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Valuation of a Firm with a Tax Loss Carryover

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ABSTRACT: This paper examines the effects of a tax loss carryover on the market and book values of a firm's assets. The loss carryover has a direct effect on market value by sheltering future income from tax, and a direct effect on book value due to the recognition of a deferred tax asset. The failure to discount the deferred tax asset to its present value causes the market-to-book ratio of the deferred tax asset to be less than 1. However, positive skewness in the distribution of future taxable income can cause the market-to-book ratio to exceed 1 because the market value depends on the mean level of future tax benefits, while the book value is based on the median level of future tax benefits. The loss carryover also has an indirect effect on firm value in that it induces the firm to exercise its real option to invest early. This reduces firm value before investment takes place and decreases the market-to-book ratio of physical assets after investment takes place.

Keywords: *deferred taxes; net operating loss carryovers; valuation; real options.*

JEL Classification: *H25.*

INTRODUCTION

A tax loss carryover is a valuable asset for a firm because it shelters some portion of the firm's future income from tax. The financial accounting system reflects a tax loss carryover as a deferred tax asset, perhaps offset by a valuation allowance. This paper characterizes the direct and indirect effects of a firm's tax loss carryover on the market value of its stock and the book value of its assets.

We first examine the direct effect of the loss carryover on firm value and characterize the market-to-book ratio of the deferred tax asset. This ratio can diverge from 1 for four reasons. First, neither the deferred tax asset nor the valuation allowance is discounted to its present value, which decreases the market-to-book ratio. Second, a valuation allowance is not established under generally accepted accounting principles (GAAP) if the probability that some of the loss carryover will expire is less than 50 percent. This also decreases the ratio for those firms without a valuation allowance, but with some positive probability of having a tax loss carryover expire. Third, the market value of the tax loss carryover reflects the mean level of future tax savings associated with the carryover, whereas the net book value reflects the median level of future tax savings. Positive (negative) skewness in the probability distribution of future income increases (decreases) the ratio. Fourth, a posi-

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tive probability of negative returns before expiration of the loss carryover increases the market-to-book ratio, even for symmetric income distributions. The option to abandon the project in the case of negative returns induces positive skewness into the distribution of realized returns. We also show that the effect of the size of the tax loss carryover on the market-to-book ratio of the deferred tax asset depends on whether the firm has a valuation allowance. The ratio is decreasing in the size of the loss for a firm without a valuation allowance; the ratio is increasing for a firm with a valuation allowance.

Next we examine the indirect effects of a loss carryover on firm value. We consider a setting in which the expenditure that creates the loss carryover also provides the firm with an opportunity to make an investment in physical capacity, so the initial expenditure provides the firm with a real option (Dixit and Pindyck 1994; Amram and Kulatilaka 1999). We show that the greater the loss carryover, the faster the firm makes the investment, which weakly decreases the value of the firm's real option. This decrease is an implicit tax associated with the tax loss carryover (Scholes et al. 2002). This implicit tax manifests itself as a decrease in the market value of the tax loss carryover prior to when the investment in capacity occurs. It also manifests itself as a decrease in the market-to-book ratio of a firm's physical assets after the investment in capacity takes place.

Our study relates to Amir et al. (2001), which argues that the deferred taxes recorded under U.S. GAAP are overstated because they are not discounted to their present value. While we agree that failure to discount causes the book value of the deferred tax asset to exceed the market value of the tax loss carryover, we identify other factors that can cause the book value to be less than the market value. Specifically, we show that when the mean tax benefits exceed the median tax benefits, the market-to-book ratio can exceed 1 by a substantial margin.

Amir et al. (1997), Ayers (1998), and Amir and Sougiannis (1999) empirically investigate the relations between market and book values using linear regression models in which stock price is explained by accounting variables. The regression coefficient on the deferred tax asset from a tax loss carryover represents the market-to-book ratio of the deferred tax asset. Amir et al. (1997), Ayers (1998), and Amir and Sougiannis (1999) each report regression coefficients on either the deferred tax asset or the valuation allowance that exceed 1. If the only difference between book value and market value relates to present value discounting, one would expect the regression coefficient to be between 0 and 1. The results of our model provide possible explanations for why regression coefficients could exceed 1.

Miller and Skinner (1998) and Schrand and Wong (2000) examine the use of the deferred tax asset valuation allowance to manage earnings and do not find evidence of earnings management; therefore, we do not consider earnings management in this study. Instead, we examine a benchmark case in which both the deferred tax asset and valuation allowance comply with the literal requirements of GAAP.

Our study also relates to the literature on the effect of tax loss carryovers on the timing and magnitude of investment decisions. Wielhouwer et al. (2000) examine the effects of a net operating loss (NOL) arising from accelerated tax depreciation on optimal investment decisions. In this paper, we focus on the effects of a NOL created by an initial tax deductible investment (e.g., R&D) on the timing of a subsequent investment in physical capacity.

The second section derives the direct effects of a tax loss carryover on firm value and on the market-to-book ratio of the deferred tax asset. The third section extends our analysis to cases following a merger in which the use of the acquired corporation's loss carryover is limited under Internal Revenue Code (IRC) § 382. The fourth section derives the effects of the tax loss carryover on the value of a real option and on the cost of investment once the real option is exercised. The fifth section summarizes the empirical implications of our results, and the last section concludes the paper.

DIRECT EFFECTS OF THE TAX LOSS CARRYOVER

In this section, we derive the market value of the firm's tax loss carryover and the book value of the associated deferred tax asset. We then characterize the market-to-book ratio of the deferred tax asset, net of any valuation allowance.

Model

The firm owns nondepreciable physical assets that will generate income in perpetuity. The stochastic rate of income is denoted Y, and a possible realization is denoted y. We assume that Y is a random variable with a probability density function $f(\cdot)$ and a cumulative density function $F(\cdot)$. We let p denote the probability that $Y > 0$, so that $Y \leq 0$ with probability $1 - p$. In the latter case, the firm decides to abandon the project, so that realized income is equal to zero. The fact that the firm has the option to abandon the project if $Y \leq 0$ implies that the market value of the firm is determined by $Y^+ = \max\{0, Y\}$. Note that Y is uncertain as of the date that the investment occurs, but is constant in the sense that once Y is realized, it does not vary over time. The assumption of constant future cash flows enhances the tractability of the model without qualitatively changing the results.

The firm faces a tax rate τ on its taxable income. The firm also has a tax *net operating loss* (*NOL*) carryover L. This loss carryover could arise due to a previous investment (e.g., in R&D) that, along with the physical assets, makes it possible to generate income. In the fourth section, we examine in detail a two-stage investment model. In this section, we simply assume the existence of a NOL and examine its market value and book value. Given a realization y of Y, taxable income is y per unit of time if there is no NOL, and zero otherwise; in the latter case, the NOL decreases at the rate of y per unit of time until it is either fully used, or until it expires on date w.

All after-tax cash flows are distributed to the shareholders as dividends as they are generated. The stock price V is equal to the present value of all future after-tax cash flows, discounted at the interest rate r. In the absence of a loss carryover ($L = 0$) the present value of future after-tax cash flows for a given y is equal to:

$$\int_0^\infty (1 - t)ye^{-rt}dt = \frac{y(1 - t)}{r}. \tag{1}$$

Therefore, the firm's stock price V_0 is:

$$V_0 = \int_0^\infty \frac{y(1 - \tau)}{r} f(y)dy = \frac{E[Y^+](1 - \tau)}{r}. \tag{2}$$

Value of the Tax Loss Carryover

In the presence of a loss carryover ($L > 0$), the present value of future after-tax cash flows depends on whether part of the loss carryover will expire unused. We distinguish between two different cases. In the first case, $L \leq wy$, which implies that the NOL is fully used before it expires. The stock price then consists of two parts. The first part is the present value of pretax cash flows earned between dates zero and L/y, at which point the NOL is fully used. The second part is the present value of future after-tax cash flows earned after date L/y. This yields for a given y:

$$V_1(y) = \int_0^{L/y} ye^{-rt}dt + \int_{L/y}^{\infty} y(1 - \tau)e^{-rt}dt$$
$$= \frac{y(1 - \tau)}{r} + \frac{\tau y(1 - e^{-rL/y})}{r}. \quad (3)$$

In the second case, $L > wy$, which implies that some of the loss L expires on date w. The stock price in this case also consists of two parts. The first part is the present value of pretax cash flows earned between date zero and w, at which point the NOL expires. The second part is the present value of future after-tax cash flows earned after date w. This yields for a given y:

$$V_2(y) = \int_0^{w} ye^{-rt}dt + \int_{w}^{\infty} y(1 - \tau)e^{-rt}dt$$
$$= \frac{y(1 - \tau)}{r} + \frac{\tau y(1 - e^{-rw})}{r}. \quad (4)$$

The value of the loss carryover on date zero reflects the possibility that some of the NOL carryover will expire on date w ($y < L/w$), and the possibility that all of the NOL carryover will be used ($y \geq L/w$). Therefore, the value reflects an average of V_2, the value when $y < L/w$, and V_1, the value when $y \geq L/w$. Because the market value of the firm is equal to zero when $y \leq 0$, we have:

$$V = \int_0^{L/w} V_2(y)f(y)dy + \int_{L/w}^{\infty} V_1(y)f(y)dy. \quad (5)$$

Substituting in the values of V_1 and V_2 from Equations (3) and (4) into Equation (5) and subtracting the value of V_0 from Equation (2) (the value of the firm when $L = 0$) yields the value of a firm's tax loss carryover, denoted VCF.

$$VCF = \frac{\tau}{r} \int_0^{L/w} y(1 - e^{-rw})f(y)dy + \int_{L/w}^{\infty} y(1 - e^{-rL/y})f(y)dy. \quad (6)$$

Deferred Tax Asset and Valuation Allowance

We now consider how L is reflected on the firm's balance sheet. The firm pays zero tax when the loss is incurred and, assuming the loss cannot be carried back, records a deferred tax asset (DTA) equal to τL, less any valuation allowance under SFAS No. 109 (FASB 1992). Because NOLs can only be carried forward a limited number of years (IRC §172(b)(1)), SFAS No. 109 requires that a valuation allowance be established under certain circumstances. Paragraph 96 reads as follows:

> The Board believes that the criterion required for measurement of a deferred tax asset should be one that produces accounting results that come closest to the expected outcome, that is, realization or nonrealization of the deferred tax asset in future years. For that reason, the Board selected more likely than not as the criterion for measurement of a deferred tax asset. Based on that criterion, (a) recognition of a deferred tax asset that is expected to be realized is required, and (b) recognition of a deferred tax asset that is not expected to be realized is prohibited.

Paragraph 97 reads in part:

> The Board intends more likely than not to mean a level of likelihood that is more than 50 percent.

Paragraph 98 reads in part:

> The Board acknowledges that future realization of a tax benefit sometimes will be expected for a portion but not all of a deferred tax asset, and that the dividing line between the two portions may be unclear. In those circumstances, application of judgment based on a careful assessment of all available evidence is required to determine the portion of a deferred tax asset for which it is more likely than not a tax benefit will not be realized.

The median y^* of the stochastic rate of income Y is defined to be the solution to:

$$F(y^*) = 1/2. \tag{7}$$

Because a valuation allowance is required if there is a greater than 50 percent probability that some of the loss L will not yield a future tax benefit, a valuation allowance must be established if $L > wy^*$, and cannot be established otherwise. In case $y^* \leq 0$, the book value of the NOL is equal to zero because the valuation allowance is equal to the deferred tax asset. Therefore, we focus on the case where $y^* > 0$, or equivalently, $p > ½$. The valuation allowance is:

$$VA = \max\{0, \tau (L - wy^*)\}. \tag{8}$$

Market-to-Book Ratio

Next, we derive the ratio of the market value of the net operating loss carryover and the book value of the NOL. We consider two types of firms that differ in the size of their NOL and in the time it takes before the NOL expires. A type-A firm is one that has not recognized a valuation allowance because $L_A \leq w_A y^*$, so $VA = 0$. A type-B firm has recognized a valuation allowance because $L_B > w_B y^*$, so $VA > 0$.

Let β_A denote the ratio of the market value of the loss carryover, VCF, to its book value for a type A firm.

$$\beta_A = \frac{VCF}{DTA}. \tag{9}$$

For a type B-firm, the market-to-book ratio equals:

$$\beta_B = \frac{VCF}{DTA - VA}. \tag{10}$$

These ratios are determined by two main effects: discounting and uncertainty. In order to isolate the discounting effect, we first consider the situation in which Y is a known constant. The market-to-book ratio then only reflects time value of money considerations.

Proposition 1: When Y is known with certainty to be y, then;

$$0 < \beta_A = \frac{y(1 - e^{-rL_A/y})}{rL_A} < 1, \quad \text{and} \tag{11}$$

$$0 < \beta_B = \frac{(1 - e^{-rw_B})}{rw_B} < 1. \tag{12}$$

Furthermore, for firms of type A ($yw_A \geq L_A$) and B ($yw_B < L_B$) with identical pretax cash flows y:

$$\beta_A > \beta_B \quad iff \quad \frac{L_A}{y} < w_B.$$

All proofs are in the appendix.

Under certainty, the future tax savings associated with L is equal to the book value of the deferred tax asset. Therefore, the term β_A diverges from 1 only because of time value of money considerations. The factors that cause β_A to diverge from 1 are the length of time it takes to realize the tax benefits (L/y) and the opportunity cost to the firm of delaying the realization of the tax benefits (r). As either the length of time it takes to realize the benefits or the interest rate approaches zero, β_A approaches 1. As was the case of β_A, β_B is between zero and 1 because the coefficient only reflects time value of money considerations. In this case, the length of time it takes to use the tax loss L reflects the remaining carryover period w instead of L/y. Proposition 1 shows that the market-to-book ratio of a firm's NOL carryover depends on the length of time that the loss carryover shelters the firm's income from tax. A firm that fully uses its loss carryover does so by date L/y, while a firm that loses part of its loss carryover uses losses until date w. The longer it takes a firm to use the loss carryover, the lower the market-to-book ratio of that loss carryover.

Example 1

Suppose $Y = y = 2$, $L_A = L_B = 30$, $w_A = 20$, and $w_B = 10$. In this case, $DTA = 30\tau$, firm A has no valuation allowance, and $VA = 10\tau$ for firm B. $\beta_A < \beta_B$ because firm B uses its asset faster than does firm A (in 10 years for B as opposed to 15 years for A). Alternatively, suppose $Y = y = 2$, $w = 20$, $L_A = 30$, and $L_B = 50$. Then $\beta_A > \beta_B$ because firm A uses its loss carryover for 15 years, whereas firm B uses its loss carryover for 20 years.

Therefore, if firms A and B have the same L but some of firm B's loss expires unused because it has a shorter carryover period w over which the NOL can be used, the market-to-book ratio of firm B is higher than that of firm A. In contrast, if firms A and B have the same w, but some of the NOL of firm B expires because it has a greater loss carryover, then the market-to-book ratio of firm A is higher. Therefore, the effect of the expiration of some of the NOL on the market-to-book ratio is theoretically ambiguous in the certainty case.

We now consider the effect of uncertainty. Substituting $DTA = \tau L$ and VCF from Equation (6) yields:

$$\beta_A = \frac{\int_0^{L/w} y(1 - e^{-rw})f(y)dy + \int_{L/w}^{\infty} y(1 - e^{-rL/y})f(y)dy}{rL}. \tag{13}$$

The difference compared to β_A from Proposition 1 is that Y is a random variable instead of a constant. Whereas β_A only reflected time value of money considerations in the certainty case, β_A in Equation (13) reflects both time value considerations and the possibility that part of the loss L expires unused. To quantify the effect of an expiring loss on β_A, we derive upper and lower bounds on β_A in absence of time value considerations, i.e., when the interest rate r equals zero.

Proposition 2: $1/2 < \lim_{r \to 0} \beta_A < p$.

The lower and upper bounds of β_A reflect the valuation allowance rules of SFAS No. 109. The probability of losing a portion of a firm's loss carryover can be as low as zero percent or as high as 50 percent without recognizing a valuation allowance. When L is sufficiently small, the probability that the tax benefit associated with L is fully used is close to p, and so β_A is close to p when L is close to zero. As L increases, β_A falls because the probability that some of the loss L will expire unused grows. This probability can be as high as 50 percent without recognizing a valuation allowance. This result shows that even in the absence of discounting, the book value of the deferred tax asset can exceed its market value.

Next, we consider the market-to-book ratio for a type-B firm, for which $VA = \tau(L - wy^*)$. Because $VA > 0$ for a type-B firm, $\beta_B = VCF/(DTA - VA)$. Substituting $DTA = \tau L$, $VA = \tau(L - wy^*)$ and VCF from Equation (6) yields:

$$\beta_B = \frac{\int_0^{L/w} y(1 - e^{-rw})f(y)dy + \int_{L/w}^{\infty} y(1 - e^{-rL/y})f(y)dy}{rwy^*}. \quad (14)$$

As was the case with β_A, β_B reflects both time value of money considerations and the difference between the expected future tax savings and the book value of the tax loss carryover. To quantify this difference, we again determine the upper and lower bounds of β_B when $r = 0$.

Proposition 3: $1/2 < \lim_{r \to 0} \beta_B < \frac{E[Y^+]}{y^*}$.

As L grows sufficiently large, the probability that some of the loss will expire converges to 1. As that happens, the market-to-book ratio β_B converges to $E[Y^+]/y^*$, which is the ratio of the expected level of future tax benefits ($\tau w \int_0^{\infty} yf(y)dy = \tau wE[Y^+]$) to the amount of future tax benefits that are reflected on the balance sheet ($DTA - VA = \tau wy^*$). The VA may overstate the expected unused portion of the loss because VA reflects the *median* unused loss while the stock price reflects the *mean* unused loss. Therefore, β_B may exceed 1. The fact that the coefficient can become greater than 1 is illustrated in the following example, where $f(y)$ has positive skew and there is zero probability on negative income.

Example 2

Let Y be lognormally distributed with a location parameter μ and dispersion parameter σ^2. Then $E[Y] = e^{\mu + \sigma^2/2}$, $y^* = e^{\mu}$, and $E[Y]/y^* = e^{\sigma^2/2}$. Because an increase in σ^2 increases $E[Y]$ but not y^*, $E[Y]/y^*$ could exceed 1 by a substantial margin.

The following example shows that the market-to-book ratio can exceed 1 even when the underlying income distribution is symmetric. This is due to the fact that the market value reflects the expected income *if positive*, which is different from the expected income without accounting for the option to abandon the project.

Example 3

Let $Y \sim N(\mu,\sigma^2)$. Then $y^* = \mu$ and $E[Y^+] > E[Y] = \mu$. In particular, $E[Y^+]$ converges to $1/\sqrt{2\pi}$ as μ converges to zero. In that case, the market-to-book ratio can become arbitrarily large.

Finally, the following example illustrates the market-to-book ratio for a discrete probability distribution.[1]

Example 4

A firm has a tax loss carryover L with a remaining life of 5 years ($w = 5$). Its future annual income prospects are 10 with probability 49 percent, 50 with probability 2 percent and 1,000 with probability 49 percent. These assumptions imply that $y^* = 50$ and $E[Y] = 496$. A valuation allowance is recognized when $L > 250$ and the market-to-book ratio becomes:

$$\beta_B = \frac{0.49 \times 50 + 0.02 \times 250 + 0.49 \times L}{250} = \frac{29.5 + 0.49 \times L}{250}.$$

For low values of L ($L \leq 450$), the expected use is lower than the median use; the opposite holds for higher values of L. This implies that depending on L, β_B can vary between 0.608 and 9.918.

Note that because the ratio $E[Y^+]/y^*$ can become substantially larger than 1, the market-to-book ratio can also exceed 1 for positive r. When $E[Y^+] > y^*$ *and* $r > 0$, the market-to-book ratio could be either greater than or less than 1, depending on the relative sizes of the two effects.

Next, we examine the effects of the parameter L on the market-to-book ratios β_A and β_B. As before, we focus on the special case in which $r = 0$ so as to distinguish between the effects of present value discounting from the differences between the book value of the deferred tax asset and the expected future tax savings associated with that asset.

Proposition 4: When $r = 0$:

(i) $\text{Lim}_{L\to 0}\beta_A = p$

(ii) $\frac{\partial \beta_A}{\partial L} < 0$

(iii) When $L = wy^*$, $\beta_A = \beta_B$

(iv) $\frac{\partial \beta_B}{\partial L} > 0$

(v) $\text{Lim}_{L\to\infty}\beta_B = \frac{E[Y^+]}{y^*}$.

Proposition 4 shows that the relation between the NOL carryover L and the market-to-book ratio β is not monotone. When L is close to zero, the probability that it will yield a tax benefit of τL is close to p, so the market-to-book ratio is close to p. As L increases, both the market and book values increase; however, the market value grows more slowly because the probability that some of the loss will expire unused increases with L; this

[1] This example is based on the example presented by Shelley Rhoades-Catanach when discussing our paper at the 2003 *JATA* Conference.

causes the market-to-book ratio to decline when $0 < L < wy^*$. When $L \geq wy^*$, the market value continues to increase with L, while the book value remains at τwy^*; this causes the market-to-book ratio to increase as L increases. As L becomes arbitrarily large, the market-to-book ratio converges to the ratio of the mean future tax savings to the median future tax savings.

EFFECTS OF THE §382 LIMITATION

Section 382 of the Internal Revenue Code limits the use of the tax loss carryover of a corporation that is acquired in a merger or stock purchase. The annual limitation is the product of the value of the acquired corporation and the long-term tax-exempt interest rate (IRC §382(b)(1)). In this section, we examine the effects of the §382 limitation on the market-to-book ratio of the NOL carryover.

We let the parameter π denote the maximum amount of loss carryover that can be used per unit of time under §382. This implies that the amount of the loss that can be used per unit of time equals:

$$Z = \min\{\pi, Y\}. \tag{15}$$

Valuation

We first consider the value of the loss carryover. There are two cases to consider. First, when $\pi w < L$, some part of the loss will expire unused at date w, because the maximum amount of loss that can be used equals $\min\{\pi, Y\}w \leq \pi w < L$. Equation (6) and the fact that the amount of the loss used per unit of time equals π when $y > \pi$ implies that:

$$VCF = \frac{\tau}{r}\int_0^{\pi} y(1 - e^{-rw})f(y)dy + \int_{\pi}^{\infty} \pi(1 - e^{-rw})f(y)dy. \tag{16}$$

Second, when $\pi w \geq L$, the level of income y will determine whether some of the loss will expire unused. When $y < L/w$, part of the loss will expire unused at date w. When $L/w < y < \pi$, all the loss will be used by date L/y. In both cases the amount of loss that is used per unit of time equals y. Finally, when $y > \pi$, all the loss will be used; but this will only happen at date $L/\pi > L/y$ because the amount that can be used per unit of time equals $\pi < y$ due to the §382 limitation. Therefore, as in Equations (6) and (16), we find the following expression for the market value of the NOL carryover:

$$\begin{aligned} VCF = {} & \frac{\tau}{r}\int_0^{L/w} y(1 - e^{-rw})f(y)dy \\ & + \frac{\tau}{r}\int_{L/w}^{\pi} y(1 - e^{-rL/y})f(y)dy + \frac{\tau}{r}\int_{\pi}^{\infty} \pi(1 - e^{-rL/\pi})f(y)dy. \end{aligned} \tag{17}$$

Valuation Allowance

Because the amount of the loss that can be used per unit of time is the stochastic variable $Z = \min\{\pi, Y\}$, SFAS No. 109 implies that the valuation allowance equals:

$$VA = \max\{0, \tau(L - wz^*)\}, \tag{18}$$

where z^* denotes the median of Z. This in turn implies $z^* = \min\{\pi, y^*\}$. Therefore, it follows that:

$$VA = \max\{0, \tau(L - wy^*)\} \quad \text{if } y^* < \pi$$
$$= \max\{0, \tau(L - w\pi)\} \quad \text{if } y^* \geq \pi,$$

so that the valuation allowance is not affected by §382 as long as either $\pi \geq y^*$ or $w\pi \geq L$. If $\pi < y^*$ and $w\pi < L$, then the §382 limitation changes the book value of the deferred tax asset by increasing the valuation allowance.

Market-to-Book Ratio

The effect of a §382 limitation on the market-to-book ratio depends on whether the limitation changes the valuation allowance. If it does not, then the limitation decreases the market value of the carryover without decreasing its book value, which causes the market-to-book ratio to decrease.

Proposition 5: If either $\pi \geq y^*$ or $\pi \geq L/w$, the §382 limitation decreases the market-to-book ratio.

Next, we consider the case in which the §382 limitation affects both the market value and the book value of the loss carryover, which occurs when $\pi < y^*$ and $\pi < L/w$. In that case, the net book value of the loss carryover is $\tau w\pi$, and the market-to-book ratio (β_C) is:

$$\beta_C = \frac{\int_0^{\pi} y(1 - e^{-rw})f(y)dy + \int_{\pi}^{\infty} \pi(1 - e^{-rw})f(y)dy}{rw\pi}. \tag{19}$$

Recall from Equations (9) and (10) that β_A and β_B denote the market-to-book ratios of type A and B firms, respectively, in the absence of the §382 limitation. The following proposition deals with the market-to-book ratio when the §382 limitation reduces the net book value of the loss carryover by increasing the valuation allowance *VA*.

Proposition 6: If $\pi < L/w$ and $\pi < y^*$, so that the §382 limitation affects both the market value and the valuation allowance, then there exists a $\tilde{\pi} \leq \min\{L/w, y^*\}$ such that:

(a) The §382 limitation increases the market-to-book ratio if:

$$\beta_A(\beta_B) < \frac{1 - e^{-rw}}{rw} p \quad \text{and} \quad \pi < \tilde{\pi}.$$

(b) The §382 limitation decreases the market-to-book ratio if:

$$\beta_A(\beta_B) \geq \frac{1 - e^{-rw}}{rw} p \quad \text{or} \quad \pi > \tilde{\pi}.$$

Proposition 6 implies that the §382 limitation can either increase or decrease the market-to-book ratio of the NOL when it affects the valuation allowance. When without the §382 limitation the market-to-book ratio is high ($>((1 - e^{-rw})/rw)p$), the reduction in market value due to the §382 limitation dominates the reduction in book value. When without the §382 limitation the market-to-book ratio is moderate ($<((1 - e^{-rw})/rw)p < 1$), the §382 limitation increases the market-to-book ratio for low values of π and decreases the market-to-book ratio for higher values of π.

Our analysis shows the effects of both discounting and uncertainty. When eliminating discounting by setting $r = 0$, we find that when the limitation creates a valuation allowance for a type A firm, then the §382 limitation always weakly increases the market-to-book ratio. This is a consequence of the fact that if $r = 0$, then $\beta_A < p$ and $\tilde{\pi} = \min\{L/w, y^*\}$. In contrast, when $r > 0$, $\tilde{\pi} < \min\{L/w, y^*\}$, so that a decrease occurs for sufficiently high values of π.

INDIRECT EFFECTS OF A TAX LOSS CARRYOVER

In this section, we examine the indirect effects of the tax loss carryover on market and book values. The carryover affects the timing of the firm's investment decision, which in turn affects the cost of the investment when it is actually made.

Model

A firm invests an amount L on date zero. This investment provides the firm with an opportunity to invest $Ke^{-\alpha T}$ on a subsequent date T.[2] The second investment generates income in perpetuity. The first expenditure can be thought of as an investment in intellectual property that provides the opportunity for the second expenditure, which can be thought of as an investment in capacity. One example of a two-stage investment is an R&D expenditure in the first stage that yields a patent, followed by the construction of physical plant to produce and sell a product using the patent. A second example is an investment in mineral exploration in the first stage, followed by mineral extraction in the second stage.[3] The parameter $\alpha \geq 0$ represents the rate at which the cost of the investment in capacity is decreasing over time due to improvements in production technology. Because the cost of physical capacity is weakly decreasing over time, the optimal way to exploit the intellectual property acquired in stage one may be to wait before making the second stage investment.

Effect of a Tax Loss Carryover on the Value of a Real Option

We consider first the optimal investment date T_0^* for a firm without a NOL carryover. The firm chooses the investment date T that maximizes the value U_0 of the real option on date zero.

$$\begin{aligned} U_0 &= e^{-rT}\left\{\int_0^{\infty}\int_T^{\infty} y(1-\tau)\, e^{-r(t-T)} f(y)dydt - Ke^{-\alpha T}\right\} \\ &= e^{-rT}\left\{\int_T^{\infty} E[Y^+](1-\tau)e^{-r(t-T)}\, dt - Ke^{-\alpha T}\right\}. \end{aligned} \tag{20}$$

Differentiation yields the following investment strategy T_0^*:

$$T_0^* = \max\left\{0, \frac{1}{\alpha}\log\left[\frac{K(\alpha + r)}{E[Y^+](1-\tau)}\right]\right\}. \tag{21}$$

Next, we consider the investment strategy of a firm with the same investment opportunity, but with a tax loss carryover of L that can only be used to offset the income generated

[2] Alternatively, this setting applies to firms with a real investment option and an existing loss carryover of size L.
[3] There is an extensive literature on real options to invest (Dixit and Pindyck 1994; Amram and Kulatilaka 1999).

by the physical investment. Firm value then consists of two components, the real option to invest and the tax loss carryover. Whether part of the loss will expire unused depends on the firm's choice of the investment date T and the realized income y. If the physical investment occurs on date $T < w$, whether the loss expires depends on the realization of y. If income y is such that $T + L/y \leq w$, then the firm will be able to use all of its loss, and the value of the firm on date zero is given by:

$$U_1(y) = e^{-rT}\left\{\int_T^{T+L/y} ye^{-r(t-T)}\,dt + \int_{T+L/y}^{\infty} y(1-\tau)e^{-r(t-T)}dt - Ke^{-aT}\right\}. \qquad (22)$$

If income y is such that $T + L/y > w$, part of the firm's loss carryover will expire on date w, then its value is given by:

$$U_2(y) = e^{-rT}\left\{\int_T^{w} ye^{-r(t-T)}\,dt + \int_{w}^{\infty} y(1-\tau)e^{-r(t-T)}\,dt - Ke^{-\alpha T}\right\}. \qquad (23)$$

The value of the firm reflects a weighted average of U_1 and U_2:

$$U = \int_0^{L/(w-T)} U_2(y)f(y)dy + \int_{L/(w-T)}^{\infty} U_1(y)f(y)dy. \qquad (24)$$

If the firm chooses an investment date $T \geq w$, then all of the loss will expire at date w, so the value of the firm equals U_0, i.e., the value of the real option for a firm without an NOL carryover. The firm chooses the optimal investment date T^* in order to maximize its market value. It can be seen easily that $Ke^{-\alpha w} < (E[Y^+](1-\tau))/(\alpha + r)$ implies that U_0 is decreasing in T for $T \geq w$, which implies that $T^* < w$. Therefore, we assume that $Ke^{-\alpha w} < (E[Y^+](1-\tau))/(\alpha + r)$. The resulting optimal investment date affects the probability that the firm will lose part of its loss carryover.

In the following proposition we show how the investment date is affected by the NOL carryover.

Proposition 7: The optimal investment time T^* is decreasing in L.

The intuition behind Proposition 7 is that the cost of waiting is increasing in L. All firms incur the opportunity cost of postponing the benefits from the investment by waiting, which is offset by the lower investment cost associated with waiting. In addition, however, firms with $L > 0$ face an additional cost of waiting because waiting postpones the use of the tax loss carryover. Therefore, firms with loss carryovers exercise their real options earlier because they have higher waiting costs, which in turn implies that the value of the real option from Equation (20) (i.e., the value of the real option excluding the value of the NOL carryover) is lower for a firm with an NOL carryover.

Effect of a Tax Loss Carryover on Investment Costs

A second indirect effect of the NOL carryover is on the market-to-book ratio of the firm's physical assets once the investment in capacity takes place on date T^*. The book value of the assets is $Ke^{-\alpha T^*}$, and so the market-to-book ratio of the physical asset on the investment date T^* is:

$$\frac{E[Y^{+}](1-\tau)}{rKe^{-\alpha T^{*}}}. \tag{25}$$

The market-to-book ratio of the physical asset is an increasing function of T^*, and so Proposition 7 implies that the market-to-book ratio of the physical assets will be highest for firms without a tax loss carryover and lower for firms with a tax loss carryover. Therefore, there is a tax-induced difference in the relation between the market and book values of the firm's physical assets depending on whether a firm had a tax loss carryover when the investment was made.

EMPIRICAL IMPLICATIONS

In this section, we summarize the empirical implications of our model. We consider first the implications that relate to the deferred tax asset and valuation allowance, then we address the indirect effects of the tax loss carryover.

Deferred Tax Asset and Valuation Allowance

The market-to-book ratio of the net deferred tax asset corresponds to the coefficient on the deferred tax asset that one would expect in a regression that associates stock price with accounting balance sheet items. We show that the properties of the market-to-book ratio depend on whether a valuation allowance is associated with the deferred tax asset. If there is no valuation allowance, then the ratio is less than 1 because (1) the deferred tax asset is not discounted to its present value and (2) there is a possibility that part of the loss carryover will expire unused. In addition, the ratio is decreasing in the size of the loss carryover.

When there is a valuation allowance, none of these relations necessarily hold. First, the market-to-book ratio could exceed 1 because the market value reflects the mean future tax savings while the book value reflects the median future tax savings. Positive skewness in the distribution of future taxable income can cause the regression coefficient on the deferred tax asset to be greater than 1. Note that even if the underlying distribution is symmetric, the distribution of future taxable income is positively skewed because the firm will exercise its option to abandon the project when $y < 0$. Second, an increase in the loss carryover increases the market-to-book ratio because an increase in the loss increases both the gross deferred tax asset and the valuation allowance. A coefficient in excess of 1 is likely for start-up firms, because these more often have a highly skewed distribution.

A §382 limitation on the use of its tax loss carryover decreases the market value of the carryover and may decrease the book value by increasing the valuation allowance. This implies an ambiguous theoretical relation between the market-to-book ratio and the presence of a §382 limitation. If either (1) the limitation does not change the valuation allowance, (2) the market-to-book ratio was already high, or (3) the maximum amount of loss carryover that can be used per unit of time under §382 is high, then the limitation decreases the market-to-book ratio. If the limitation increases the valuation allowance, then the market-to-book ratio was already low, and the maximum amount that can be used under the limitation is low, then the limitation increases the market-to-book ratio.

Indirect Effects

The fourth section of this paper showed that the real option to invest held by a firm with a loss carryover is worth less than the real option of a firm without a loss carryover because the loss carryover induces the firm to exercise its option early. This implies that the valuation effects of a tax loss carryover are not limited to the reduction in future taxes;

to the extent the tax loss carryover distorts the firm's investment decisions, there is a countervailing implicit tax effect that reduces firm value.

The fourth section also showed how a tax loss carryover affects the market-to-book ratio of a firm's physical assets. Because a firm with a loss carryover will exercise its real option earlier than a firm without a loss carryover, we expect the loss firm to have lower market-to-book ratios with respect to its assets (other than deferred tax assets) than a similar firm without a loss carryover.

CONCLUSIONS

This paper examines the direct and indirect effects of a tax loss carryover on a firm's value. It also examines how these effects are reflected in the market-to-book ratios of the firm's assets.

A tax loss carryover has a direct effect on firm value by sheltering some portion of the firm's future income from tax. The carryover is recognized by the financial accounting system as a deferred tax asset, offset by a valuation allowance if the probability that part of the loss will go unused exceeds 50 percent. If the firm does not recognize a valuation allowance, then the market-to-book ratio of the deferred tax asset is less than 1. However, this ratio can exceed 1 when a valuation allowance is present because the market value of the loss depends on the mean future tax savings, whereas the book value of the deferred tax asset depends on the median future tax savings. This provides a possible explanation for why regression coefficients on deferred tax assets associated with tax loss carryovers sometimes exceed 1.

The loss carryover also has an indirect effect on firm value when the firm has a real option to invest. A firm with a loss carryover has a higher opportunity cost of waiting to invest and thus exercises its real option sooner than a firm without a loss carryover. Prior to the date of the investment, this indirect effect will manifest itself as a decrease in the value of the firm's real options. Following the investment, it will manifest itself as a lower market-to-book ratio of the firm's physical assets.

APPENDIX

Proof of Proposition 1

When Y is known to be y, a type-A firm is one for which $y \geq L/w$ and a type-B firm is one for which $y < L/w$. Equations (6), (8), and (9) imply that $\beta_A = y(1 - e^{-rL/y})/rL$; Equations (6), (8), and (10) imply that $\beta_B = (1 - e^{-rw})/rw$. Furthermore, the definitions of type-A and type-B firms imply that $(L_A/w_A) \leq y < (L_B/w_B)$. The ratios β_A and β_B are both of the form $(1 - e^{-z})/z$, where $z = rL_A/y$ for firm A and $z = rw_B$ for firm B. The expression $(1 - e^{-z})/z$ is decreasing in z, which implies that $\beta_A > \beta_B$ if and only if $L_A/y < w_B$.

Finally, we need to prove that the coefficients β_A and β_B are between 0 and 1. The function $(1 - e^{-z})/z$ is positive for positive values of z, and less than 1. The latter follows from the fact that $e^{-z} > 1 - z$. ■

Proof of Proposition 2

Applying L'Hopital's rule to Equation (13) and evaluating it at $r = 0$ yields:

$$\beta_A = \frac{w}{L}\int_0^{L/w} yf(y)dy + \int_{L/w}^{\infty} f(y)dy. \tag{A.1}$$

Differentiating β_A with respect to L indicates that when $r = 0$, β_A is decreasing in L, since:

$$\frac{\partial \beta_A}{\partial L} = -\frac{w \int_0^{L/w} y f(y) dy}{L^2} < 0. \tag{A.2}$$

For any type-A firm, $0 \leq L \leq wy^*$. Once again applying L'Hopital's rule shows that, when $r = 0$, β_A approaches $\int_0^\infty f(y)dy = p$ as L approaches 0, which yields the upper bound of $\beta_A = p$. Substituting $L = wy^*$ into Equation (A.1) yields:

$$\beta_A = \int_0^{y^*} \frac{y}{y^*} f(y)dy + \int_{y^*}^\infty f(y)dy.$$

The first term could be arbitrarily close to 0; the second term equals $1 - F(y^*) = \frac{1}{2}$, and thus the lower bound of β_A is 1/2. ■

Proof of Proposition 3

Applying L'Hopital's rule to Equation (14) and evaluating it at $r = 0$ yields:

$$\beta_B = \frac{w \int_0^{L/w} y f(y) dy + L \int_{L/w}^\infty f(y)dy}{wy^*}. \tag{A.3}$$

Differentiating β_B with respect to L yields:

$$\frac{\partial \beta_B}{\partial L} = \frac{\int_{L/w}^\infty f(y)dy}{wy^*} > 0, \tag{A.4}$$

so that when $r = 0$, β_B is increasing in L. For any type-B firm, $L > wy^*$. Substituting $L = wy^*$ into Equation (A.3) yields:

$$\beta_B = \int_0^{y^*} \frac{y}{y^*} f(y)dy + \int_{y^*}^\infty f(y)dy.$$

The first term could be arbitrarily close to 0; the second term equals $1 - F(y^*) = \frac{1}{2}$, and thus the lower bound of β_B is 1/2. Applying L'Hopital's rule shows that when $r = 0$, β_B converges to $\int_0^\infty y/y^* f(y)dy$ as L approaches infinity, so the upper bound of β_B is $E[Y^+]/y^*$. ■

Proof of Proposition 4

When $0 < L \leq wy^*$, $VA = 0$ and thus the market-to-book ratio is β_A. When $L > wy^*$, $VA > 0$ and thus the market-to-book ratio is β_B. The statements (i)–(v) immediately follow from the proofs of Propositions 2 and 3. ■

Proof of Proposition 5

There are two cases to consider. If $\pi < L/w$, then the effect of the §382 limitation on *VCF* is equal to the difference between Equations (6) and (16). Differentiating Equation (16) with respect to π yields:

$$\frac{\partial VCF}{\partial \pi} = \frac{\tau(1 - e^{-rw}) \int_{\pi}^{\infty} f(y)dy}{r} > 0. \tag{A.5}$$

Therefore, we need only show that Equation (6) exceeds Equation (16) when $\pi = L/w$. Subtracting Equation (16) from Equation (6) and setting $\pi = L/w$ yields:

$$\int_{L/w}^{\infty} \left[\frac{y(1 - e^{-rL/y})}{rL} - \frac{(1 - e^{-rw})}{rw}\right] f(y)dy > 0. \tag{A.6}$$

If $\pi \geq L/w$, then the effect of the §382 limitation on *VCF* is equal to the difference between Equations (6) and (17). Differentiating Equation (17) with respect to π yields:

$$\frac{\partial VCF}{\partial \pi} = \frac{\tau\left(1 - e^{-rL/\pi} - \frac{rLe^{-rL/\pi}}{\pi}\right) \int_{\pi}^{\infty} f(y)dy}{r} > 0. \tag{A.7}$$

Furthermore, Equation (17) converges to Equation (6) as π approaches infinity, and thus Equation (6) exceeds Equation (17) for all finite values of π. ■

Proof of Proposition 6

First, notice that β_C is decreasing in π on the interval $[0,\min\{L/w, y^*\}]$, since:

$$\frac{\partial \beta_C}{\partial \pi} = -\frac{1}{rw\pi^2} \int_0^{\pi} y(1 - e^{-rw})f(y)dy < 0. \tag{A.8}$$

When π approaches 0, one finds by applying L'Hopital's rule:

$$\begin{aligned} \lim_{\pi \to 0} \beta_C &= \frac{\lim_{\pi \to 0} \int_0^{\pi} y(1 - e^{-rw})f(y)dy}{rw} + \frac{(1 - e^{-rw})}{rw} \int_0^{\infty} f(y)dy \\ &= \frac{(1 - e^{-rw})}{rw} p \end{aligned} \tag{A.9}$$

Now we show that when $L/w < y^*$, it holds that $\lim_{\pi \to L/w} \beta_C < \beta_A$. For $\pi = L/w$ one has:

$$\beta_C = \int_0^{L/w} \frac{y}{rL}(1 - e^{-rw})f(y)dy + \int_{L/w}^{\infty} \frac{1}{rw}(1 - e^{-rw})f(y)dy, \tag{A.10}$$

so that:

$$\beta_C - \beta_A = \int_{L/w}^{\infty} \frac{1}{rw}(1 - e^{-rw})f(y)dy - \int_{L/w}^{\infty} \frac{y}{rL}(1 - e^{-rL/y})f(y)dy \leq 0 \qquad \text{(A.11)}$$

with a strict inequality for $r > 0$. Similarly one can show that when $\pi = y^* < L/w$ it holds that $\beta_C \leq \beta_B$, with a strict inequality for $r > 0$. Therefore, the §382 limitation causes the value of β_C to decrease from an upper bound of $((1 - e^{-rw})/rw)p$ (when π is close to 0) to a level strictly less than (for $r > 0$) or equal to (for $r = 0$) β_A (β_B) as π approaches $\min\{L/w, y^*\}$.

Therefore, there exists a $\tilde{\pi} \leq \min\{L/w, y^*\}$, such that $\beta_C > \beta_A$ (β_B) for all $\pi < \tilde{\pi}$ and $\beta_C < \beta_A$ (β_B) for all $\pi > \tilde{\pi}$. For $r = 0$, it holds that $\tilde{\pi} = \min\{L/w, y^*\}$. ■

Proof of Proposition 7

Setting the derivative of U with respect to T equal to 0 yields the following equation:

$$E[Y^+] - \tau \int_{L/(w-T)}^{\infty} ye^{-rL/y} f(y)dy = (\alpha + r)Ke^{-\alpha T}. \qquad \text{(A.12)}$$

Let us denote t^* for the solution of (A.12). Now since U is strictly concave in T on the interval $[0,w]$, and since $Ke^{-\alpha w} < (E[Y^+](1 - \tau))/(\alpha + r)$ implies that $t^* \leq w$, it follows that $T^* = \max\{0,t^*\}$.

The left-hand side of (A.12) is increasing in L and increasing in T, and the right-hand side is decreasing in T. Therefore it follows immediately that t^* is decreasing in L. Consequently, T^* is also decreasing in L. ■

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